The influence of the Coriolis force on rivers and the Baer law. Historical review

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Abstract

The Coriolis force acts due to the rotation of the Earth about its axis and deflects objects with inertial motion on the Earth's surface to the right in the northern hemisphere and to the left in the southern one. The Baer law states that the rotation of the Earth about its axis deflects rivers along the meridian to the right in the northern hemisphere and to the left in the southern one, and deflection does not exist if the motion is directed along the lines of latitude. The relationships between the Coriolis force and Baer law consist in the common cause (Earth's rotation) and consequence (deflection); however, they differ with respect to the evaluation of the effect from the direction of the inertial motion: Baer regards this direction as being decisive (i.e. deflection is maximal along the meridian and zero along the latitudes); from the viewpoint of the Coriolis force, however, the direction does not play any role.

Because of the obvious relationships between the Coriolis force and Baer law the fundamental difference in evaluation of the role of the direction of the motion has frequently been left out of consideration and for a century these two were thought to be equal. Hence the debate about the origins of the Baer law and the discussion of the problem approximately simultaneously have led to the following conclusions:

1. In the discussion at the Paris Academy in 1859 it became clear that from the physical and mathematical point of view the Baer law erroneously postulated the dependence of the deflection from the direction of motion.

2. In the first part of Baer's article, which made his law known to Europe, he formulated and presented arguments underpinning the law, whereas in the third part of the same article – influenced by the discussion in Paris – he significantly modified it. Here, he did not reject deflection due to motion along the latitude, but now believed this motion to be weaker than that along the meridian.

As can be seen, references to the Baer law in its original form were anachronistic from the very beginning.

Opinions about the effect of the Earth's rotation upon rivers are very diverse – ranging from complete compliance up to the complete rejection. That is why data for verifying and disproving the effect have been closely investigated. It has been found that – first of all in works of Russian scientists – the asymmetry of river valleys confirms the influence of the Coriolis force in very large regions; nevertheless, objections appeared mainly in two fields. On the one hand, mathematical calculations resulted in very small absolute values for the deflective force; on the other hand, attempts were made to explain existing deflections in alternative ways. An overview of the abundant literature resulted in the conclusion that all the mathematical calculations (which examined the Coriolis force not separately but in comparison with other effects) confirmed that the influence of the Earth's rotation cannot be neglected. Alternative ideas can only be valid for particular cases but do not explain the global effect, which is clear from the results of regional mapping of river asymmetry.

Introduction

In West Siberia where gigantic rivers(i.e. Ob, Yennisey and their tributaries) flow to the north, travellers in the XVIII[th] century had already noticed that the right banks of the rivers were systematically higher and steeper than the left ones. SLOVTSOV (1827) first raised the question of whether this phenomenon could possibly be related to the Earth's rotation about its axis; he expressed the view that the matter could be solved by making comparisons with other rivers of the world. His work remained unknown in Europe for many decades. It is also hard to access Slovtsov's work in Russia, so even Russian scientists usually refer to BERG's (1949, pp. 306–315) work. In the 1850s Baer, an ethnic German scientist born in Estonia and a Russian citizen, knowing but not citing Slovtsov's work, drew attention to the phenomenon that due to the Earth's rotation rivers flowing north- or southward are deflected in the northern hemisphere to the right, and in the southern hemisphere to the left. His first articles (BAER 1856a, b, c[1], 1857, 1858) were published in Russian and they are even lesser-known than the work of Slovtsov. An important component of his views was that the deflective force influences rivers, which flow along meridians, but not those that flow along latitudes.

However, on the 31 October 1859 session of the Academy of Sciences in Paris, BABINET (1859a) — inspired by PERROT's (1859) experiment[2] and based on Foucault's concept[3], developed a view (using examples from the big Siberian rivers, the Nile, and several European rivers, including the Danube) that the deflection is independent of the course of the rivers.

On the 7 November 1859 session of the same Academy, BERTRAND (1859a) expressed the view that deflection of rivers along the meridian is understandable but that of the rivers along the latitude is not. Besides, based on a mathematical formula, he claimed that the effect is too weak to deflect rivers. BABINET (1859b) stressed that the force is independent of the direction of motion and defended his view by calculating a concrete value.

On the 14 November session BERTRAND (1859b) detailed his standpoint. BABINET (1859c) replied by means of a mathematical deduction for the force deflecting rivers. Following this, DELAUNEY (1859) called attention to the fact that the deflective force had in fact been precisely formulated by Coriolis and, presenting the Coriolis's formula (equal to Babinet's one), joined Bertrand's opinion on the too small value of the force in question to deflect rivers. In his reply to Delauney, BERTRAND (1859c) pointed out that he was not willing to discuss the Coriolis force but thought it important to draw attention to the claim that its influence upon rivers was so weak that it could be neglected.

[1] Article in Astrakhanskie Gubernskie Vedomosti, 5[th] October 1856. Inaccessible, reviewed by BERG (1949).

[2] Water in a big circular pot, when moving to the sinkhole in the middle, flows by dextral slewing, which is a new confirmation of the Earth's rotation and of the influence of this rotation upon moving fluids.

[3] Foucault's well-known pendulum shows the deflective force of the Earth's rotation.

On the 21 November 1859 session BABINET (1859d) outlined in detail consequences from Foucault's and Perrot's experiments, and gave a mathematical deduction for the independence of the effect from the Earth's rotation from the direction of the motion. In the four-week dispute COMBES (1859) was the last to express an opinion; however, although he supported Bertrand's and Delaunay's view he did not want to comment on Coriolis's theorem.

So, in the Paris dispute Babinet's concept about the independence of the deflective force due to the Earth's rotation was not attacked fiercely but his opinion about the deflection of rivers by the Earth's rotation was not supported. It is worth mentioning that Babinet's argumentation based on concrete observations on concrete rivers was left out of consideration.

Participants in the dispute did not mention Baer's name (although they might possibly have heard about it) and it is unclear whether they were familiar with his views or not. (In his autobiography published in 1864 Baer — according to BERG 1949, pp. 306–315 — wrote that they were). In any case, in the dispute Bertrand essentially stood for Baer's view. In the next year BAER (1860) repeated and summarised the contents of his articles in Russian and complemented them by reflection on the Paris dispute.

The work Baer published in 1860 consists of three separate parts. These parts appeared in the same volume of the Bulletin of the Sankt-Petersburg Imperial Academy of Sciences under the same title. The first part (columns 1–49) outlines Baer's original concept. The second part (columns 218–250) gives an overview — based on the maps and travellers' descriptions that were at Baer's disposal — of rivers in Europe, Asia, North Africa, North and South America. He shows that almost everywhere the right bank is higher and steeper in the northern hemisphere, while in the southern hemishpere it is the left bank which is steeper. These two parts are mostly texts from earlier Russian publications (mainly BAER 1857 and 1858). In the third part (columns 325–382) — having been influenced by the Paris debate — Baer accepted that the deflective force also acts along the lines of latitude but emphasised that the effect is stronger along the meridian.

Based on the title of his article, later on — probably first in the work of SUESS (1863) — Baer's original concept — according to which the latitudinal rivers are not deflected due to the Earth's rotation — was referred to as the "Baer law". Baer, however, had essentially given up the restriction concerning the direction of motion in the third part of the same article (written in German), although he had outlined it in the first part. As is apparent from the comments of the readers (and not speaking about those who only refer to the article) it seems they did not reach this third part. Probably this was the reason why "Baer law" was still being cited even a century later — e.g. in TŐRY (1952), SCHMIDT (1957), PÉCSI (1959), BULLA (1964), LOYDA, PODRACKÝ (1979) and BRÁZDIL, MÁCA (1982).

In the 20[th] century the Baer law was frequently related to the Coriolis force, albeit in two different versions. In the first of these the Baer law is presented as having the same consequences as the Coriolis force — i.e. it is independent of the direction of flow. This standpoint, based on insufficient knowledge about the Baer law, was supported, for example, by Einstein (1926), Quraishy (1943), Macdonald (1952), Gabriel et al. (1957), Gerenchuk (1960), Zemtsov (1973), Matschinski (1966) and Gábris et al. (1998). On the other hand, the second opinion reflects insufficient understanding of the Coriolis force: e.g. in Schmidt (1957), Új Magyar Lexikon (1959: Baer [p. 218] and Coriolis-erő [Coriolis force, p. 468]), Bolshaya Sovetskaya Entsiklopediya (1971) Bera zakon [Baer law, p. 568], 1973: Koriolisa sila [Coriolis force, p. 561]), Loyda, Podracký (1979) and Brázdil, Máca (1982).

After Baer (1860) I only know of Müürsepp (1996) who had discovered and analysed the difference and relationships between the Coriolis force (Baer refers to it as a "formula") and the original Baer law. His work, however, was never referred to. Nevertheless, Müürsepp did not mention that Baer *de facto* had already given up his own original law in 1860; furthermore, Müürsepp's outlining of the story was not precise in some other aspects, too. It is this latter point, along with some other factors, that inspired me to compile this article. An important one of the "other factors" is the fact that most of my colleagues seem to be in a state of confusion over the Coriolis force and the Baer law. In the third part of this study I want to take into consideration the rather various views on the influence of the Coriolis force upon the rivers.

Comparison of the Coriolis force and Baer law

I only know wordy explanations of the Baer law and have not yet met its graphic expression. The Baer law can be outlined briefly as follows: on the rivers flowing *approximately along meridians,* the right bank in the northern hemisphere, and the left bank in the southern hemisphere is higher and steeper. According to Baer's explanation a river flowing along the meridian crosses areas with different linear rotation velocity but due to the inertia it tries to keep the original velocity. In the northern hemisphere the linear rotation velocity decreases from the south to the north and increases from the north to the south. A river "tries" to keep its original velocity, which is greater when it flows to the north and smaller when it flows to the south. Since the Earth rotates from west to east, a river, which flows to the north deviates in this (eastern) direction; that flowing to the south, in an opposite (western) direction — i.e. in both cases, to the right (Figure 1). It is easy to realise that in the southern hemisphere the deviation is opposite, i.e. to the left.

In a case in which the river course is different from the meridian, the deflective force — which is obviously perpendicular to the river course — is of a lesser degree (Figure 2). The lesser the degree, the bigger is the angle between the river course and the meridian. The effect disappears along lines of latitude.

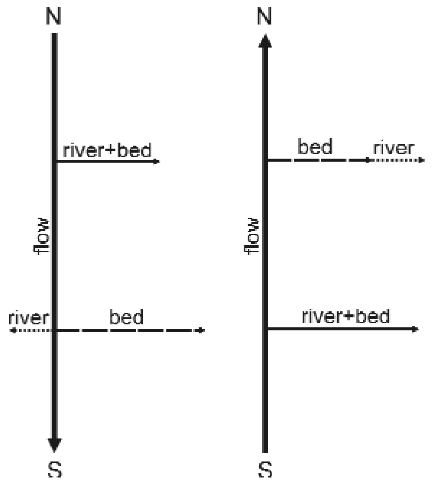

Figure 1. The Baer law for a meridianal river

Northern hemisphere. Direction and velocity of motions on the rotating Earth for an external observer: river+bed = direction and velocity of motion of the stream and bed together in the starting point, bed = factual direction and velocity of motion of the bed fixed to the solid Earth in the new point, river = direction and velocity of the virtual inertial motion of the stream during the motion towards the new point

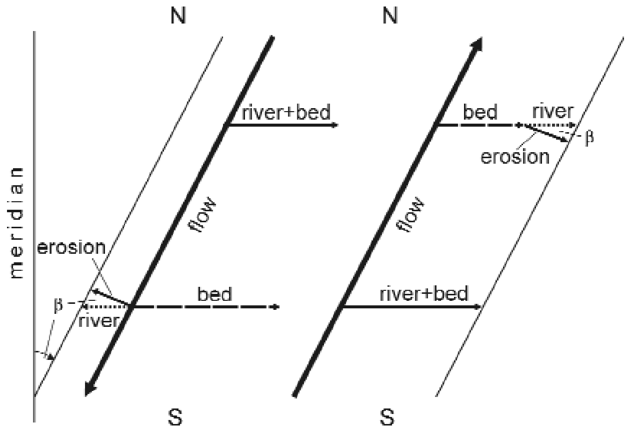

Figure 2. The Baer law for an oblique river

Northern hemisphere. Erosion = component of the inertial motion of the stream directed towards the bank, β = angle between the flow and the meridian (azimuth). For further captions, see Figure 1

The ideas outlined above were apparently so simple that they could be accepted without drawings. Consequently, when the question of the Coriolis force was also related to the Earth's rotation, it appeared more and more frequently in connection with river migration and many scientists automatically extended the dependence of the deflection from the direction of motion onto the Coriolis force itself (see above). This, however, is a fundamental physical misunderstanding: the Coriolis force *does not* depend on the direction of the motion. This is clearly seen from the formula for the Coriolis force for any point on the Earth's surface, in which the direction of motion is not included:

$$F_c = 2m \cdot v \cdot \omega \cdot \sin\varphi \text{ and } a_c = 2v \cdot \omega \cdot \sin\varphi ,$$

where F_c = Coriolis force (its horizontal component at the given point), a_c = Coriolis acceleration (its horizontal component at the given point), m = mass of the moving object,

v = linear (horizontal) velocity of the moving object at the given point, ω = angular velocity of the Earth rotation at the given point, φ = geographical latitude of the given point.

Both the Coriolis force and Coriolis acceleration are perpendicular (in a horizontal plane) to the direction of motion.

However, one thing that has to be pointed out is that physics and mathematics differ completely with regard to the respective "common sense" approaches. These two are not always in harmony but physical laws can be "warned off" from this fact. So, for example, no problem is generated in realising that the Earth circulates around the Sun and not conversely, although our everyday experience seems to contradict this knowledge. Inasmuch as, however, in early school-days we are convinced by various methods that this is the perfect knowledge and not the opposite, we accept it without any special mathematical–astronomical argumentation.

The same cannot be said about the Coriolis force. To say it as delicately as possible, the situation, in which we have no doubts concerning its influence — and especially its independence from the direction of motion — has not been reached yet. Furthermore, physicists themselves (e.g. PERSSON 2005) admit that they indeed have not made sufficient efforts to eliminate this situation.

That is why — from mere necessity — I will present here some considerations. I think there is no special need for realising that in the case of rivers along meridians (Figure 3, a) the situation is similar to that postulated by the Baer law. As for the rivers along lines of latitude (Figure 3, b), the deficiency of the Baer law can be detected: it does not take into account that, due to the curvature of the lines of latitude, rivers almost parallel to them also deviate from their original direction. The deviation also is rightward in the northern hemisphere and leftward in the southern hemisphere, as in case of rivers along meridians. In other words, the explanation of the Baer law takes into account the curvature of the Earth's surface only in planes of meridians, whereas it regards lines of latitude to be straight *ad infinitum.* The fundamental error of the explanation of Baer law consists in its neglecting of the fact that lines of latitude are circular, not straight.

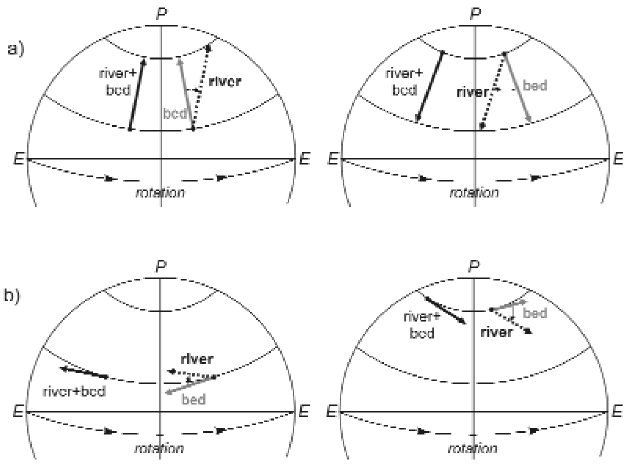

Figure 3. The effect of the Coriolis force on rivers along meridians or latitudes

Northern hemisphere. Rotation = direction of the Earth's rotation, for further captions, see Figure 1. P = pole, E = equator

With respect to this, two questions arise:

1. In the case of rivers along meridians, does the Baer law give a quantitatively correct explanation for the deflection? In other words, does it lead to the same result as the Coriolis' formula?

2. Is it true that rivers along meridians are deflected more frequently and/or more strongly than those along lines of latitude?

The first question is a purely mathematical one and it will be discussed first. The second question already leads to the principal question as to whether the Coriolis force influences rivers at all or not, and this will be discussed in the next chapter.

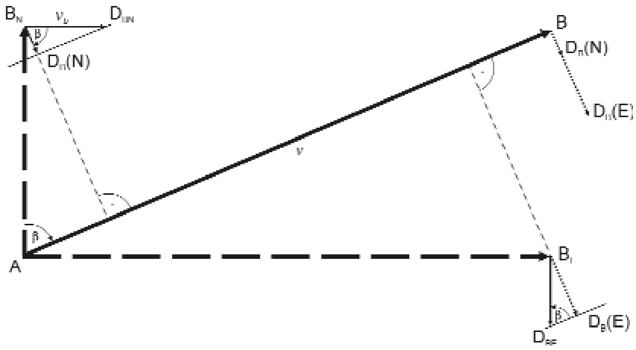

Figure 4. Sketch for the deflection of an oblique motion based on the description by BABINET (1859d)

v = AB = velocity of the motion, β = angle between the flow and the meridian (azimuth), AB_N = northern component of the velocity, AB_E = eastern component of the velocity, B_ND_{BN} = deflection of the northern component, B_ED_{BE} = deflection of the eastern component, $B_ND_{B(N)}$ = deflection of the northern component perpendicularly to the AB (= v_b = deflective velocity from the Baer Law), $B_ED_{B(E)}$ = deflection of the eastern component perpendicularly to the AB

The motion along a meridian can be defined from the formulae given by Babinet for the deflection of an oblique motion (Figure 4). Here

$$AB_N = AB\cdot\cos\beta = v\cdot\cos\beta\,,$$

$$B_ND_{BN} = AB_N\,\omega\cdot\sin\varphi = v\cdot\cos\beta\,\omega\cdot\sin\varphi\,,$$

$$B_ND_B(N) = B_ND_{BN}\cdot\cos\beta = v\cdot\cos\beta\,\omega\cdot\sin\varphi\cdot\cos\beta = \\ = v\cdot\omega\cdot\sin\varphi\cdot\cos^2\beta\,,$$

$$B_ND_B(N) = B_ND_{BN}\cdot\cos\beta = v\cdot\cos\beta\,\omega\cdot\sin\varphi\cdot\cos\beta = \\ = v\cdot\omega\cdot\sin\varphi\cdot\cos^2\beta\,.$$

Because

$$B_ND_{BN} = v_b$$

and

$$a = 2v/t\,,$$

furthermore

$$a_b = 2v\cdot\omega\cdot\sin\varphi\cdot\cos^2\beta = a_c\cdot\cos^2\beta$$

and

$$F_b = 2m\cdot v\cdot\omega\cdot\sin\varphi\cdot\cos^2\beta = F_c\cdot\cos^2\beta\,,$$

where a_b = acceleration from the Baer law, F_b = force generating this acceleration, b = azimuth of the river course, all the others are the same, as in the case of the Coriolis force and acceleration.

Besides:

$$AB_E = AB \cdot \sin\beta = v \cdot \sin\beta \, ,$$

$$B_E D_{BE} = AB_E \, \omega \cdot \sin\varphi = v \cdot \sin\beta \, \omega \cdot \sin\varphi$$

and

$$B_E D_B(E) = B_E D_{BE} \cdot \sin\beta = v \cdot \sin\beta \, \omega \cdot \sin\varphi \cdot \sin\beta = \\ = v \cdot \omega \cdot \sin\varphi \cdot \sin^2\beta \, .$$

The deflection of the northern and eastern components perpendicularly to the direction AB gives the total deflection and

$$B_N D_B(N) + B_E D_B(E) = v \cdot \omega \cdot \sin\varphi \cdot \cos^2\beta + v \cdot \omega \cdot \sin\varphi \cdot \sin^2\beta = \\ = v \cdot \omega \cdot \sin\varphi \cdot (\cos^2\beta + \sin^2\beta) = v \cdot \omega \cdot \sin\varphi \, ,$$

the result is that the direction of motion does not influence the deflective force.

As a consequence, the general formula for the Baer's acceleration is

$$a_b = 2v \cdot \omega \cdot \sin\varphi \cdot \cos^2\beta \, .$$

Thus for the motion along the meridian (cos0° = 1) the Baer law gives the value of the horizontal component of the Coriolis force; however, for a direction different from the meridian it gives a value which contains the azimuth of the course of the river (β, see Figure 2). It can be stated that the modifying coefficient is $\cos^2\beta$, and its application will result in erroneous values (along latitudes cos90° = 0, thus $a_b = 0$).

Views on the influence of the Coriolis force upon rivers

The Coriolis force does not only influence rivers. It is generally accepted that it plays a significant role in the origin of atmospheric and oceanic currents, and numerous human activites and events (cannon fire, hammer throw in athletics, derailments etc.). The fact that I only discuss rivers does not mean that I restrict the Coriolis effect solely to them. Here only an outline is given of the framework for the discussion.

The Coriolis force appears on rotating objects, and deflects other objects in an inertial motion on rotating objects. It should be understood that this motion, is apparent: when observing from the rigid frame of the rotating object the inertial motion is straight; it only appears to be curved for observers *inside* the rotating object. As a consequence, the Coriolis effect takes place on all the objects with inertial motion on the Earth's surface. The motion, which is straight for extraterrestrial observers, is curved for observers on the Earth.

The Coriolis force on the Earth's surface can be unfolded into two components: a local horizontal (i.e. in the plane tangential to the Earth's surface) and a local vertical (i.e. perpendicular to the previous one). The horizontal component can be detected and measured by the Foucault pendulum whereas the vertical component represents the Eötvös effect, which also can be measured. Consequently, not only the existence of the Coriolis force cannot be disclaimed but neither can the validity of the formula which describes it. (In an opposite way, the concept of the Foucault pendulum and the Eötvös effect equally prove the Earth's rotation). If the horizontal component of the Coriolis force depends on the direction of motion, the Foucault pendulum would rotate (or not rotate) depending on the direction of the initial impulse. Experiments, however, show that the Foucault pendulum — totally independent of the direction of the initial impulse — rotates (up to complete circle or more) until the time it stops moving.

In the case of the rivers, it is not the existence of the Coriolis force which raises any question but the measure of its influence. The problem has been examined by many scientists in many aspects and this gives me opportunity to provide something like a classification. I think the analysis of three particular aspects seems reasonable. I name the first aspect "empiric". Investigations which have aimed at an estimation of the frequency of the deflection of rivers can be placed in the empiric aspect.. I think it is the most important aspect since it concerns the historical origin of the problem and the fundamental feature of all the sciences — that is, *facts*. The second aspect I refer to as the "mathematic" one. The mathematic aspect includes considerations and calculations which are aimed at an estimation of the value of the Coriolis force in case of rivers and which have tried to draw conclusions. The analyses of the alternatives to the above I have grouped into the third aspect.

Of course, these three aspects rarely appear alone and this means the above classification concerns ideas, not articles and other works. Nevertheless, distinguishing between the aspects is useful since it helps in understanding which views originated from insufficient knowledge and which can be taken into account for final conclusions.

On the empiric base

Construction of the empiric base had already been started by Baer when he collected and interpreted information on the Earth's rivers. (In contrast to Baer, Babinet worked with a limited database). From the present-day point of view this was a great achievement at a time when for many areas even maps were unavailable and conclusions could only be drawn from travellers' descriptions.

Since that time quantitative data have appeared for large areas of Eurasia. For example, VOSKRESENSKIY (1947) and GERENCHUK (1960) have produced data on the Russian Plain, and ZEMTSOV (1973) has done the same for the West Siberian Plain. ZHUKOVSKIY (1970), surveying the northern part of East Siberia, mapped the asymmetry of river valleys and concluded that dextral asymmetry prevails strongly everywhere. However, figures have only been published by

VOSKRESENSKIY (1947) and ZEMTSOV (1973). Voskresenskiy stated that more than 90% of asymmetrical valleys show dextral asymmetry. Zemtsov estimated the rate of dextral asymmetry to be 70-75% for all the valleys. I have no information about the mapping of the asymmetry of enormous areas in other parts of the world but the published maps cover more or less continuously a minimum of 10%[4] of the continents — i.e. they are quite representative.

From the mapping performed it was concluded that the prevalence of the dextral asymmetry reflects the Coriolis effect, whereas the sinistral (or variable along or between the rivers) asymmetry can be related to local features, mostly tectonic tilts. This means, at the same time, that the dextral asymmetry cannot also everywhere be related to the Coriolis effect: in several areas the two effects could be combined and could intensify each other.

From the West Siberian data it is unclear what the ratio of the valleys without asymmetry is, but it was obviously above zero. Perhaps it does not represent a large error if it is supposed that there are 50-50% valleys with sinistral asymmetry and with no asymmetry, respectively. In that case sinistral asymmetry would be characteristic of 12-15% of all the river valleys, and the ratio of the valleys with dextral asymmetry among the asymmetrical valleys would be 83-85%, which is close to the data from European Russia.

If, starting from the idea that 85-90% of asymmetrical valleys reveal dextral asymmetry, for the rest of the 10-15% — primarily tectonic — origins can be supposed. Since for large areas the tectonic effects must be regarded as accidental relative to river courses, a similar ratio (10-15%) of tectonic effects seems to be valid in valleys with dextral asymmetry. Consequently, 70-80% of all the asymmetrical valleys are exclusively related to the Coriolis force. It can be said that this is the global empirical probability for its pure existence.

At the same time, it is also clear that the influence of the Coriolis force upon a concrete section of a concrete river cannot be confirmed by investigations of this type. Thus the empiric method only provides a statistical basis but is incapable of evaluating individual cases.

On the mathematical calculations

The goal of mathematical calculations was to define the measure of the (horizontal component) of the Coriolis force acting on rivers.

Such calculations had already been performed by the participants in the Paris dispute (BERTRAND 1859a, BABINET 1859b, d, COMBES 1859); they got very small values. Since none of the later calculations brought any change in this field, the demonstration of two results seem to be enough.

From the formula in ZÖPPRITZ (1882) with respect to latitude 50° under the influence of the Coriolis force on the right bank of a 1 km wide river with 1 m/s flow velocity, the water table elevates by 1 cm as compared to the left bank. (Zöppritz thought that this effect can be compensated by some tufts of grass)

Almost a century later LOYDA, PODRACKÝ (1979) computed the value of the Coriolis effect for various latitudes and flow velocities. For exampe, at 50° latitude and a 1 m/s flow velocity they registered a deflective force of 110 mN (mN = Milli-Newton —1N = 0,102 kp = 1 kg·m/s^2, 110 mN = 110 g·m/s^2).

These and analogous calculations convinced many scientists to accept the opinion first expressed by BERTRAND (1859a), DELAUNEY (1859) and COMBES (1859) about the Coriolis force being too weak to deflect rivers (see for example, GÁBRIS et al. 1998). The common deficiency of evaluations of this type lies in the fact that they give a subjective opinion and declare that the force is “too small”; yet they do not provide any essential arguments for thinking so.

Calculations of this type can be regarded as “absolute” since their results, on the one hand, are direct consequences coming from the Coriolis’s formula; on the other hand, these stand alone — with no comparisons. Although in both cases it has been mentioned that the effect is much weaker than that from the wind, this is not a convincing argument (the wind effect will be dealt with in next chapter). For example, weathering takes place under the influence of: sun radiation and heating, water and the carbon-dioxide content of the air, and several other factors. Yet if one of the latter seems to be weaker than the others, it would be rash to neglect this in the conclusion. Probably GERENCHUK (1960) and MYURSEPP (1976) are right: in absence of limit even a small force can produce significant changes if acts for a long time.

And now let me turn to the results of those calculations which quantitatively compared the Coriolis effect with the other effect. They can be referred to as “relative”. Calculations of this type have been performed by numerous authors. It seems to be sufficient to mention only some of them. EAKIN (1910) found that on the Mississippi at Columbus (33° 30’ northern latitude) the potential lateral effect from the Coriolis force is 18% higher on the right curve than on the left. LAKSHA, HUDYAKOV (1968), using similar calculations, concluded that in West Siberia the ratio of the acceleration toward the right and left curves on the Tara River[5] (57° n.l.) is 1.52 at flood and 1.42 at low water; on the Vasyugan River[6] the equivalent figures are 1.74 and 1.63. In other words, the acceleration (force) towards the right banks is roughly one and a half times stronger on right curves than on the left ones. The difference between the two estimations (i.e. the American and Russian) is in accordance with the latitude differences: sin 33° = 0.54, sin 58° = 0.85, 0.85/0.54 = 1.56, and (1.42÷1.74)/1.18 = 1.20÷1.47.

SHANTZER (1951) compared the Coriolis effect with the component of the gravity field oriented along the flow (see also VOSKRESENSKIY 1947) and with the centrifugal force in

[4] When taking into account that in large areas of the continents (deserts and ice-covered regions) rivers are totally absent, a much bigger rate can be derived.

[5] A 806 km long right tributary of the Irtysh River with an approximately E-W direction.

[6] A 1062 km long left tributary of the Ob River with an approximately S-N direction in its upper and W-E in its lower sections.

Table 1. Comparison of various forces acting on the water of the River Volga at Kazan (55° 50’ n.l.) compiled from data by SHANTZER (1951, pp. 199–200)

Force	Acceleration [10^[illegible] m/s²]		Flow velocity [m/s]		Inclination along the flow	
	low water	flood	low water	flood	low water	flood
Coriolis	3.6	36.2				
Gravity	49	6686	0.3	3	10''	1''
Centrifugal, R – 2 km	4.5	450	0.3	3		
Centrifugal, R 8 km	1.125	112.5	0.3	3		

* = component along the flow, R = radius of the curve

river curves (Table 1). Shantzer himself pointed out that during a flood the Coriolis effect is about 3/4 of the gravity component at low water and that the centrifugal and Coriolis forces are quite comparable. Nobody disclaims permanent undercutting of the outer banks of river curves due to the centrifugal forces. Consequently there is no base for denying the significant role of the Coriolis force in the same process.

Also, a “relative” calculation was performed by MATSCHINSKI (1966), who elaborated a specific algorithm to analyse the symmetry of river curves. Analysing curves of the Seine and Tevere rivers he estimated that they differ from the symmetric case by 15-17% (Seine) and 8-10% (Tevere), hence the influence of the Coriolis force is obvious.

It can be seen that while the interpretation of the results of “absolute” calculations are totally subjective, the “relative” mathematical calculations confirm a quite distinct influence of the Coriolis force upon the asymmetrical undercutting of right and left curves which are responsible for the deflection of rivers.

On the alternative explanations

Alternative explanations may be of various types (and it should be pointed out that they did not appeared in the 1859 Paris discussion — see above). Some of the explanations accepted Baer’s first argumentation (BAER 1856a, b): the feature in question is so widespread that it requires a uniform explanation, while some other did not.

STEFANOVIĆ VON VILOVO (1881) listed inhomogeneous weathering and roundness, dominant wind orientation and related sand blow as the factors of asymmetrical undercutting of river banks. He estimated the migration of the main rivers in Hungary as 0.47 m/year for the Danube and as 0.31 m/year for the Tisza, and assumed that this is due to the dominant south-eastern wind direction during the spring floods. In his argumentation the inhomogeneous weathering and roundness are obviously local factors, hence they cannot have a general role. The sand blow in turn is a secondary result of the wind and it has no independent role. The only remaining — perhaps general factor — is the wind. His concept obviously did not require uniform explanation for the asymmetry.

ZÖPPRITZ (1882) explained the migration of the Siberian rivers flowing towards the north by referring to the dominant (in his opinion) western wind. Similar dextral asymmetry of the Volga River flowing towards the south was explained in his concept by geological (albeit not specified) factors. KÖPPEN (1890a) supported ZÖPPRITZ’s (1882) opinion concerning the large Siberian rivers but assumed that in Southern Russia in cold, stormy periods the wind is dominantly eastern, and that is why the right banks of the rivers flowing towards the south is steep. Some of his contempories accepted his argumentation (for an overview, see KÖPPEN 1890b), but others did not. Both of them agreed with Baer in the need for a uniform explanation, but they offered another one. KÖPPEN (1890a) even mentioned that the dominant wind is generated by the Earth’s rotation, which in this way influences rivers indirectly.

Later on, however (for example, see the overview by NEMÉNYI 1952) the wind disappeared from the discussions and argumentations — obviously, it became clear that the real wind orientation, even in the cases mentioned above, significantly differs from that assumed, and the wind cannot give a global explanation.

As another alternative explanation the aspect (i.e. of exposure) appeared, from the very beginning, for local cases only (see for example BRYAN, MASON 1932, VOSKRESENSKIY 1947, LAKSHA, HUDYAKOV 1968, GÁBRIS 2007). This is because measurement of the processes influencing the slopes depends on numerous climatic factors, and although these play a role over large areas they vary from one climatic zone to the other.

Tectonic hypotheses were for a long time the most widespread. However, as alternatives of the Coriolis force they obviously reflected the opinions of those scientists who did not accept Baer’s view on the need for a uniform explanation (see above). Tectonic hypotheses mostly stressed the influence of faults. Two versions of this idea can be outlined. The first of them was restricted to the statement that rivers flow along faults, and opposite banks (i.e. the flanks of the fault) have emerged at different levels. The other version considered the asymmetry in a wider framework and tried to systematise the displacements upon the faults by assuming that blocks between the faults have been tilted uniformly over large areas (see for example GERENCHUK 1960, LAKSHA, HUDYAKOV 1968, ZHUKOVSKIY 1970, ZEMTSOV 1973). In this way some kind of regionalism was introduced into the explanation of the asymmetry; however, a global explanation of course could not be reached. (The authors referred to did not aspire to this since they accepted the dominant role of the Coriolis force).

As has been seen, the tectonic hypotheses, which made attempts to provide an alternative to the Coriolis effect, essentially did not accept the fact that the undercutting of right banks in the northern (and of left banks in the southern) hemisphere is so widespread that it needs a global explanation (BAER 1856a, b, 1860, ZÖPPRITZ 1882, KÖPPEN 1890a). So the tectonic hypotheses hardly did not even refer to the global character of the feature; yet it is obvious that on a hemispheric scale any tectonic phenomenon would result in an approximately equal frequency of dextral or sinistral undercutting and cannot result in the significant dominance of one of them.

Conclusion

The empiric base and "relative" mathematical calculations do not allow alternative hypotheses to disclaim the influence of the Coriolis force upon rivers. On the other side, nobody insists on the absence of different forces and features, and not even on their permanently subordinate role as compared to the Coriolis force. That is why the influence of the Coriolis force upon rivers undoubtedly exists but is not exclusive.

Summary

In the light of both statistical (i.e. empiric) investigations and mathematical calculations, the influence of the Coriolis force upon rivers is an undoubted fact. The Baer law in turn is erroneous in its original formulation since it postulates that the effect coming from the Earth's rotation depends on the direction of motion. This contradicts, for example,the well-known experiments with the Foucault pendulum. Hence it is time to discontinue references to it, especially as the author almost immediately cancelled his "law".

Mapping of the river valley asymmetry in the Russian and West Siberian Plains resulted in the conclusion that the Coriolis force is exclusively or mainly responsible for the asymmetry of 40-50% of all river valleys; in other cases the role of tectonic tilts can also (!) be confirmed or supposed. In 85-90% of the asymmetrical valleys the asymmetry corresponds to that which would be expected from the Coriolis force. (In one sixth of these cases the two — i.e. tectonic and Coriolis — forces act in the same direction.) As a consequence, in all the cases in which the asymmetry corresponds to that expectable from the Coriolis effect, the latter must be taken into account. The cases of opposite asymmetry in about a sixth of all cases cannot be used to disprove the Coriolis effect in general. Asymmetry of a valley with a probability of about 70-80% arises under the influence of the Coriolis force.

It should be taken into account that mainly two features are responsible for the asymmetry of river valleys: tectonic tilting and the Coriolis force. In general the effect of the latter is frequently much stronger rather than weaker than that of the tectonic tilts. The two forces can act in the same or in opposite directions. The factual asymmetry depends on the direction of the forces and on their measure relative to each other, which can be and must be studied in each concrete case.

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References

BABINET, J. 1859a: Influence du mouvement de rotation de la terre sur le cours de rivière. – *Compte rendu des séances de l'académie des sciences* 49 (31 octobre 1859.), pp. 638–641.

BABINET, J. 1859b: Réponse. – *Compte rendu des séances de l'académie des sciences* 49 (7 novembre 1859.), p. 659.

BABINET, J. 1859c: Sur le déplacement vers le nord ou vers le sud d'un mobile qui se meut librement dans une direction perpendiculaire au méridien. – *Compte rendu des séances de l'académie des sciences* (14 novembre 1859.), pp. 686–688.

BABINET, J. 1859d: Démonstration de la loi de M. Foucault sur la tendance transversale d'un point qui se déplace à la surface de la terre. – Évaluation de la force qui produit dans le rivières la tendance à la érosion des rives. – *Compte rendu des séances de l'académie des sciences* 49 (21 novembre 1859.), pp. 769–775.

BAER, [K. E.][7] 1856a: O kaspiyskom rybolovstve. Statya IV[8] (in Russian: On the Caspian fishery. Article IV). – Z*hurnal Ministerstva gosudarstvennykh imushchestv* 58 (1), pp. 1–28[9].

BAER, [K. E.] 1856b: Peremeshchenie Volgi s vostoka na zapad (in Russian, translated title: Shift of Volga from the east to the west). – *Vestnik Estestvennykh nauk* 3, pp. 219–224.

BAER, [K. E.] 1857: Pochemu u nashikh rek, tekushchikh na sever ili na yug, pravyy bereg vysok, a levyj nizmen (in Russian, translated title: Why ours rivers, flowing to the north or to the south, have high right and low left banks). – *Morskoy Sbornik* 27 (1), pp. 110–126.

BAER, [K. E.] 1858: Dopolnenie k statye: Pochemu u nashikh rek, tekushchikh na sever ili na yug, pravyy bereg vysok, a levyj nizmen (in Russian, translated title: Complement to the article:

[7] Hereinafter first names are not indicated in the original. (Itt és tovább a keresztnevek nincsnek megadva az eredetiben.)

[8] See item 6) on pp. 24–28. (L. a 6) pontot a 24–28. oldalon.)

[9] No continuous page numeration in the volume. (A kötetben nincs folyamatos oldalszámozás.)

Why our rivers, flowing to the north or to the south, have high right and low left banks). – *Morskoy Sbornik* 28 (5), pp. 83–103.

BAER, K. E. v. 1860: Über ein allgemeines Gesetz in der Gestaltung der Flußbetten. – *Bulletin de l'académie impériale des sciences de St-Pétersbourg* 2, columns[10] 1–50, 218–250, 353–382.

BERG, L. S. 1949: *Ocherki po istorii russkikh geograficheskikh otkrytiy* (in Russian, translated title: Articles on the history of Russian geographical discoveries). – Izdanie Akademii nauk SSSR, Moskva-Leningrad, 465 p.

BERTRAND, J. 1859a: Note relative à l'influence de la rotation de la terre sur la direction des cours d'eau. – *Compte rendu des séances de l'académie des sciences* 49 (14 novembre 1859.), pp. 658–659.

BERTRAND, J. 1859b: Seconde Note sur l'influence du mouvement de la terre. – *Compte rendu des séances de l'académie des sciences* 49 (14 novembre 1859.), pp. 685–686.

BERTRAND, J. 1859c: Réponse à M. Delauney. – *Compte rendu des séances de l'académie des sciences* 49 (14 novembre 1859.), p. 692.

Bolshaya Sovetskaya Enciklopediya (in Russian, translated title: Great Soviet Encyclopaedia) 1971: 4. Brasos–Vesh. – Izdatelstvo «Sovetskaya Enciklopediya», Moskva, 600 p.

Bolshaya Sovetskaya Enciklopediya (in Russian, translated title: Great Soviet Encyclopaedia) 1973: 13. Konda–Kun. – Izdatelstvo «Sovetskaya Enciklopediya», Moskva, 608 p.

BRÁZDIL, R., MÁCA, B. 1982: Coriolisova síla a její důsledky v krajinné sfére Země (in Slovak with English summary: Coriolis force and its consequences in the landscape sphere of the Earth). – *Geografický Časopis* 34 (1), pp. 34–49.

BRYAN, K., MASON, S. L. 1932: Asymmetric valleys and climatic boundaries. – *Science* 75 (1939), pp. 215–216.

BULLA B. 1964: *Magyarország természetföldrajza* (in Hungarian, translated title: Geography of Hungary). – Tankönyvkiadó, Budapest, 420 p.

COMBES, M. 1859: Observations au sujet de la communication de M. Perrot et de la Note de M. Babinet qui l'accompagne. – *Compte rendu des séances de l'académie des sciences* 49 (21 novembre 1859.), pp. 775–780.

DELAUNAY, M. 1859: Observations sur la mȩme question. – *Compte rendu des séances de l'académie des sciences* 49 (14 novembre 1859.), pp. 688–692.

EAKIN, H. M. 1910: The influence of the Earth's rotation upon the lateral erosion of streams. – *Journal of Geology* 18 (5), pp. 435–447.

EINSTEIN, A. 1926: Die Ursache der Mäanderbildung der Flußläufe und des sogenannten Baerschen Gesetzes. – *Die Naturwissenschaften* 11, pp. 223–224.

GABRIEL, V. G., BROCKMAN, L., HABER, D. F. 1957: Influence of Coriolis force on river profiles. – *World Oil* 145 (4), p. 89.

GÁBRIS GY. 2007: *Földfelszín és éghajlat* (in Hungarian, translated title: Earth's surface and climate). – ELTE Eötvös Kiadó, Budapest, 224.

GÁBRIS GY., MARIK M., SZABÓ J. 1998: Csillagászati földrajz (in Hungarian, translated title: Astronomic geography). – Nemzeti Tankönyvkiadó, Budapest, 338 p.

GERENCHUK, K. I. 1960: *Tektonicheskie zakonomernosti v orografii i rechnoy seti Russkoj ravnini* (in Russian, translated title: Tectonic regularities in orography and river network of the Russian Plain). – Izdanie Lvovskogo universiteta, Lvov (Zapiski Geograficheskogo obshchestva SSSR, novaya seriya 20), 242 p.

KÖPPEN, W. 1890a: Die vorherrschenden Winde und das Baer'sche Gesetz der Flussbretten. – *Meteorologische Zeitschrift* 7 (1), pp. 34–35.

KÖPPEN, W. 1890b: Die vorherrschenden Winde und das Baer'sche Gesetz der Flussbretten. – *Meteorologische Zeitschrift* 7 (5), pp. 180–183.

LAKSHA, B. G., HUDYAKOV, G. I. 1968: *O prichine asimmetrii rechnykh stokov (na primere Ob-Irtyshskogo mezhdurechya)* (in Russian, translated title: On the cause of the asymmetry of fluvial streams [on the example of the Ob-Irtysh interfluve]). – Izvestiya Akademii nauk SSSR, seriya geograficheskaya 3, pp. 76–83.

LOYDA, L., PODRACKÝ, P. 1979: Coriolisova síla a Baerův zakon (in Czech with English abstract: The Coriolis force and the Baer's law and with German summary: Die Corioliskraft und das Baergesetz). – *Sborník Československé Geografické Společnosti* 84 (4), pp. 303–310.

MACDONALD, J. E. 1952: The Coriolis effect. – *Scientific American* 1952 (5), pp. 72–78.

MATSCHINSKI, M. 1966: Les formes des boucles des fleuves et la force de Coriolis. – *Comptes rendus hebdomadaires des seances de l'Académie des sciences, D - Sciences Naturelles* 262 (19), pp. 2001–2004.

MÜÜRSEPP, P. 1996: Über die Bildung der Flußbetten. Das Baer–Babinetsche Gesetz. – *Jahrbuch der Akademischen Gesellschaft für Deutschbaltischen Kultur in Tartu (Dorpat)* 1 (1989–1994, 1995)[11]. http://www.aai.ee/abks/11.html

MYURSEPP, P. V. 1976: Zakon Bera–Babine (in Russian, translated title: The Baer–Babinet Law). – *Konferentsiya, posvyashchennaya pamyati Bera, 30/IX-2/X 197/6, Tartuskiy Gosudarstvennyj universitet, Institut zoologii i botaniki, Akademiya nauk Estonskoj SSR, Tartu,* p. 52.

NEMÉNYI, P. F. 1952: Annotated and illustrated bibliographic material on the morphology of rivers. – *Bulletin of the Geological Society of America* 63 (6), pp. 595–644.

PÉCSI M. 1959: *A magyarországi Duna-völgy kialakulása és felszínalaktana* (in Hungarian, translated title: The generation and morphology of the Danube Valley in Hungary). – Földrajzi Monográfiák 3, Akadémiai Kiadó, Budapest, 346 p.

PERROT, M. 1859: Nouvelle expérience pour rendre manifeste le mouvement de rotation de la terre. – *Compte rendu des séances de l'académie des sciences* 49 (31 octobre 1859.), pp. 637–638.

PERSSON, A. O. 2005: The Coriolis effect: Four centuries of conflict between common sense and mathematics, Part I: A history to 1885. – *History of Meteorology* 2, pp. 1–24.

QURAISHY, M. S. 1943: River meandering and the Earth's rotation. – *Current Science* (= *Proceedings of the Indian Academy of Science, Section A*) 12 (October), p. 278.

SCHMIDT E. R. 1957: *Geomechanika* (in Hungarian, translated title: Geomechanics). – Akadémiai Kiadó, Budapest, 275 p.

SHANTZER, E. V. 1951: *Allyuvij ravninnykh rek umerennogo poyasa i ego znachenie dlya poznaniya zakonomernostey stroeniya i formirovaniya allyuvialnykh svit* (in Russian, translated title: Fluvial deposits of the rivers in the plains of moderate climate belt and its significance for understanding the constitution and genesis of the fluvial sequences). – Trudy Instituta geologicheskikh nauk Akademii nauk SSSR 135, Geologicheskaya seriya 55, 274 p.

STEFANOVIĆ von VILOVO, J. R. 1881: Ueber das seitliche Rücken der Flüsse. – *Mittheilungen der kais. und kön. Geographischen*

[10] No page numeration, each column of two columns per page is numbered. (Nincs oldalszámozás, az oldalankénti két oszlop mindegyikének folyamatos számozása van.)

[11] No page numeration. (Nincs oldalszámozás.)

Gesellschaft in Wien, 24 (der neuen Folge 14), pp. 167–187.

SUESS, ED. 1863: Ueber den Lauf der Donau. – *Oesterreichische Revue* 4, pp. 1–11.

TŐRY K. 1952: *A Duna és szabályozása* (in Hungarian, translated title: The Danube and its regulation). – Akadémiai Kiadó, Budapest, 454 p.

Új Magyar Lexikon (in Hungarian, translated title: New Hungarian Encyclopaedia) 1959: 1. A–C. – Akadémiai Kiadó, Budapest, 536 p.

VOSKRESENSKIY, S. S. 1947: Asimmetriya sklonov rechnykh dolin na territorii Evropeyskoy chasti SSSR (in Russian, translated title: Asymmetry of slopes of the river valleys in the European part of the USSR). – *Voprosy geografii* 4, *Glyatsiologiya i geomorfologiya,* pp. 107–114.

ZEMTSOV, A. A. 1973: Asimmetriya rechnykh dolin Zapadno-Sibirskoy ravniny (in Russian, translated title: Asymmetry of river valleys of the West Siberian Plain). – *Izvestiya Vsesoyuznogo Geograficheskogo obshchestva* 105 (2), pp. 142–148.

ZHUKOVSKIY, YU. S. 1970: Erozionno-akkumulyativnyy relief basseyna r. Oleneka i levoberezhya nizhney Leny (in Russian, translated title: Erosional-depositional relief of the Olenek River basin and the left bank of the lower Lena). – *Izvestiya Vsesoyuznogo Geograficheskogo* 102 (1), pp. 10–17.

ZÖPPRITZ, K. 1882: Über den angeblichen Einfluss der Erdrotation auf die Gestaltung von Flussbetten. – *Verhandlungen des Deutschen Geographischentagen zu Halle* am 12., 14. und 15. April 1882, 2, pp. 47–53.